Strategic Facts

Grade Level: 1st-5th

Skills: Basic addition, subtraction, multiplication, and division facts; thinking strategies

Groups: Whole class or individual lessons

Materials: 100 two sides (red/yellow) counters per group
Paper and crayons
Basic addition/subtraction table
Basic multiplication/division table
Deck of cards

Introduction: Learning to attack unknown basic facts using thinking strategies will insure accuracy and ultimately enhance speed. Helping students understand and use the following thinking strategies for the basic addition, subtraction, multiplication, and division facts is recommended as a prelude to instant recall.

Directions:
Have each student use their two-sided counters to model each strategy. Then use crayons to draw a picture of the model and record a related number sentence. After each strategy is introduced, discuss a verbalization of the strategy.

ADDITION THINKING STRATEGIES

Adding Zero

Model	Illustration	Number Sentence
one red counter plus no yellow counters	●	1 + 0 = 1
two red counters plus no yellow counters	● ●	2 + 0 = 2
three red counters plus no yellow counters	● ● ●	3 + 0 = 3
four red counters plus no yellow counters	● ● ● ●	4 + 0 = 4

Focus on the emerging pattern. Predict the results for adding nothing (zero) to 5, 6, and 7. Complete the chart to verify. Have students describe in their own words what they see. For example, *adding zero (nothing) means the amount stays the same.* Generate a similar chart when the first addend is zero (0 + 1 = 1, 0 + 2 = 2, 0 + 3 = 3, etc.). Ask, *does it matter where the zero is added?*

Adding One

Model	Illustration	Number Sentence
one red counter plus one yellow counter	● ○	1 + 1 = 2
two red counters plus one yellow counter	● ● ○	2 + 1 = 3
three red counters plus one yellow counter	● ● ● ○	3 + 1 = 4
four red counters plus one yellow counter	● ● ● ● ○	4 + 1 = 5

Focus on the emerging pattern. Predict the results for adding "one more" to 5, 6, and 7. Complete the chart to verify. Have students describe in their own words what they see. For example, *adding one means the amount increases to the next counting number.* Generate a similar chart when the first addend is one (1 + 1 = 2, 1 + 2 = 3, 1 + 3 = 4, etc.). Ask, *does it matter where the one is added?*

Doubles

Model	Illustration	Number Sentence
two red counters plus two yellow counters	● ● ○ ○	2 + 2 = 4
three red counters plus three yellow counters	● ● ● ○ ○ ○	3 + 3 = 6
four red counters plus four yellow counters	● ● ○ ○ ● ● ○ ○	4 + 4 = 8
five red counters plus five yellow counters	● ● ● ○ ○ ○ ● ● ○ ○	5 + 5 = 10

Have students discuss what they notice about the **doubles.** Some possible responses are: *They're easy* (This can be attributed to the rhythmic verbal pattern.); *both sets or addends are the same, it's like multiplying by two; all the sums are even; etc.* Continue the discussion as long as acceptable ideas are generated.

Near Doubles

Model	Illustration	Number Sentence
six red counters plus six yellow counters	● ● ● ○ ○ ○ ● ● ● ○ ○ ○	6 + 6 = 12
six red counters plus seven yellow counters	● ● ● ○ ○ ○ ○ ● ● ● ○ ○ ○	6 + 7 = 13
six red counters plus five yellow counters	● ● ● ○ ○ ○ ● ● ● ○ ○	6 + 5 = 11

Repeat the above process of one more and one less for several other doubles. Try adding one or removing one from the first set (addend) instead of the second. Have students verbalize ways to use the **near doubles** strategy. Using 8 + 7 = [ ] as an example, some possible strategy applications are:

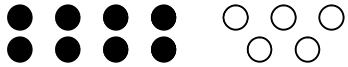
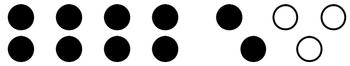
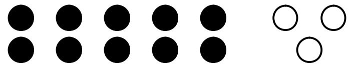
8 + 7 = [ ] 8 + 8 = 16 8 + 7 is one less So 8 + 7 = 15	eight and eight are 16 so 1 less is 15	8 + 7 = [ ] 7 + 7 = 14 8 + 7 is one more So 8 + 7 = 15	seven & seven are 14, 1 more is 15.

Counting On

Model	Illustration	Number Sentence
five red counters plus two yellow counters	● ● ● ● ● ○ ○ say 5… 6 ..7	5 + 2 = 7
six red counters plus three yellow counters	● ● ● ● ● ● ○ ○ ○ say 6.. 7.. 8.. 9	6 + 3 = 9
three red counters plus four yellow counters	● ● ● ○ ○ ○ ○ 5.. 6.. 7 ← say 4	3 + 4 = 7
two red counters plus seven yellow counters	● ● ○ ○ ○ ○ ○ ○ ○ 8.. 9 ← say 7	2 + 7 = 9

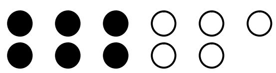
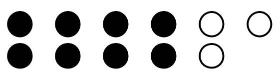
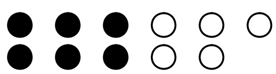
The **counting on** strategy is most effectively used when one of the addends is 2 or 3. The most efficient application is to start with the larger addend and count on. Have students verbalize in their own words how to use this strategy. For example, *look for a 2 or 3, then count on from the other number. If I see 2 + 9 = [ ], say 9 and then 10, 11.*

Making Ten

Model	Illustration	Number Sentence
eight red counters plus five yellow counters		8 + 5 = [ ]
make ten red counters by flipping over two yellow counters		(8+ 2) + (5-2) =
ten red counters plus three yellow counters		10+ 3 = 13

Making Ten is most effectively used when one of the addends is an 8 or 9 because then only 1 or 2 more makes ten. Of course, students must know the combinations for 10 in order to use this strategy. Have students verbalize in their own words how to use this strategy. For example, *5 + 9 means more less and one more for 4 + 10, which is 14.*

Finding/Making a Ten

Model	Illustration	Number Sentence
six red counters plus five yellow counters		6 + 4 = 10 so add 1 = 11
eight red counters plus three yellow counters		8 + 2 = 10 so add 1 =11
six red counters plus five yellow counters		6 + 5 = 11

Pretend a Nine is a Ten

This strategy works great for 9 + 5 = [ ], 3 + 9 = [ ], etc. Students pretend that the nine is a ten, 10 + 5 = 15, then backup one number, 15…14; 3 + 10 = 13, then backup one number 13..12. Students need to have mastered the facts for ten.

SUBTRACTION THINKING STRATEGIES

Related Combinations

Use related addition facts to figure out a subtraction fact. For example: 13 – 7 = [ ];
think 7 + [?] = 13; 7 + 6 = 13, so 13 – 7 = 6 OR 7 + 7 = 14, so 7 + 6 = 13, so 13 – 7 = 6.

Subtracting Zero

Observe the pattern of subtracting zero. When nothing (zero) is subtracted, the difference is always the same amount. For example: 17 – 0 = 17, 3 – 0 = 3, 14 – 0 = 14, etc.

Subtracting One
Observe the pattern of subtracting one. When one is subtracted, the difference is always one less. For example: 17 – 1 = 16; 3 – 1 = 2; 14 – 1 = 13; 6 – 1 = 5, etc.

Doubles
The rhythmic nature of the addition doubles should be used for solving related subtraction facts. For example: 18 – 9= [ ]; think 9 + 9 = 18; so 18 – 9 = 9.

Counting Back
When the number to be subtracted is 2 or 3, the counting back strategy can be effectively used. For example: 12 – 3 = [ ]; think 12…11…10…9; so 12 – 3 = 9.

Adding On
When the difference is 2 or 3, the adding on strategy can be effectively used. For example:
9 – 7 = [ ]; think "How much more than 7 is 9?" 2 more; so 9 – 7 = 2.

MULTIPLICATION THINKING STRATEGIES

Multiplying by Zero
Observe the pattern and meaning of multiplying by zero. A factor of zero always gives a product of zero. For example: 6 X 0 = [ ]; means six sets of nothing (zero), so 6 X 0 = 0; or 0 X 6 = [ ], means no (zero) sets of 6, so 0 X 6 = 0.

Multiplying by One
Observe the pattern and meaning of multiplying by one. A factor of one always gives the other factor as the product. For example: 5 X 1 = [ ], means five sets of one, so 5 X 1 = 5; or
1 X 5 = [ ], means one set of five, so 1 X 5 = 5.

Skip Counting
This strategy can first be introduced with the multiplies of 2, 5, and 10. Then other multiplies can be applied if students learn to skip count by 3's, 4's, 6's, 7's, etc. For example: 3 X 5 = [ ], think *count by fives three times. 5…10…15; so 3 X 5 = 15.*

Splitting a Factor
This strategy can be introduced once expertise with factors less than 5 is evident. Split a larger factor into two smaller factors. Then apply the distributive property of multiplication.
For example: 8 X 3 = (4 + 4) X 3 = (4 X 3) + (4 X 3) = 12 + 12 = 24.

Nines Pattern
Observe the pattern when one of the factors is 9.

1 X 9 = 9
2 X 9 = 18 Look at the product.

3 X 9 = 27 The tens digits is one less than the other factor.
4 X 9 = 36 The sum of the digits is 9.
5 X 9 = 45
6 X 9 = 54
etc.

Repeated Addition
This strategy is most efficient when one factor is less than 5. Change the multiplication fact into a "column" addition problem. For example: 4 X 6 = [ ] means four groups of six; think 6 + 6 + 6 + 6 = 24; so 4 X 6 = 24.

Commutative Property
Observe different examples of the commutative property. For example: a 3 by 5 array and a 5 by 3 array; 3 sets of 5 and 5 sets of 3; 3 X 5 and 5 X 3 on the calculator. The answer to both pairs is always the same i.e., 15. The pictures are different. This reduces memorization of the multiplication facts in half.

<u>DIVISION THINKING STRATEGIES</u>

Related Combinations
Use related multiplication facts to figure out a division fact. For example: 48 ÷ 6 = [ ];
think 6 X [?] = 48, 6 X 8 = 48, so 48 ÷ 6 = 8.

Repeated Subtraction
This strategy is most efficient when skill with mental subtraction is evident. Change the division fact into a subtraction problem. Keep subtracting out the amount of divisor until the answer is zero. Count how many times the divisor was subtracted for the answer to the division problem. For example: 12 ÷ 4 = [ ]; think 12 – 4 = 8, 8 – 4 = 4, 4 – 4 = 0; 4 was subtracted 3 times, so 12 ÷ 4 = 3.

Skip Counting Backwards
This strategy works most efficiently if skill with forward skip counting is evident first. Skip count backwards by multiples of the divisor until the answer is zero. Count how many skips were made for the answer to be zero. For example: 12 ÷ 4 = [ ]; think 12..8..4..0, so 12 ÷ 4 = 3.